

# NATURAL CONVECTIVE HEAT TRANSFER IN AN ENCLOSURE PARTLY FILLED WITH A NON-POROUS INSULATION

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## ABSTRACT

Solves steady, laminar, two-dimensional, conjugate natural convection in an rectangular enclosure numerically. The enclosure consists of heated and cooled isothermal walls connected by either adiabatic or perfectly conducting end walls. The enclosure is partially filled with a finitely conducting non-porous thermal insulation, adjacent to the heated surface. Solves the governing equations (in stream function-vorticity form) using a finite element method. Obtains data  $Pr = 0.7$  over a Rayleigh number range (based on the enclosure width) of  $0 \leq Ra \leq 10^6$ . The results show the effect of solid insulation thickness on the average Nusselt number for a range of enclosure aspect ratios, inclination angles and solid-to-fluid conductivity ratios. Aims to determine the conditions that produce the minimum overall heat transfer rate.

KEY WORDS      Natural convection    Conjugate    Enclosure    Thermal insulation

## NOMENCLATURE

$A$  = Aspect ratio,  $H/W$   
 $A_{cor}$  = Aspect ratio in correlation (24)  
 $C_p$  = Constant pressure specific heat  
 $Gr$  = Grashof number  
 $g$  = Gravitational acceleration  
 $H$  = Height of the enclosure  
 $k$  = Thermal conductivity  
 $k_r$  = Solid-to-fluid conductivity ratio,  $k_s/k_f$   
 $Nu_c$  = Cold wall average Nusselt number  
 $Nu_{cor}$  = Average Nusselt number from correlation (24)  
 $Nu_h$  = Hot wall average Nusselt number  
 $Pr$  = Fluid Prandtl number  
 $p$  = Pressure  
 $Q$  = Total heat transfer rate  
 $Ra$  = Rayleigh number,  $GrPr$   
 $Ra_{cor}$  = Correlation Rayleigh number given by (25)  
 $S$  = Thickness of the solid insulation  
 $T$  = Temperature  
 $u, v$  = Velocity components in the  $x$  and  $y$  directions  
 $W$  = Width of the enclosure

$X, Y$  = Dimensionless Cartesian co-ordinates  
 $x, y$  = Cartesian co-ordinates

### Greek symbols

$\beta$  = Coefficient of thermal expansion of the fluid  
 $\theta$  = Dimensionless temperature  
 $\theta_i$  = Dimensionless solid-fluid interface temperature  
 $\nu$  = Kinematic viscosity  
 $\rho$  = Fluid density  
 $\psi, \Psi$  = Stream function and dimensionless stream function  
 $\omega, \Omega$  = Vorticity and dimensionless vorticity  
 $\Phi$  = Enclosure inclination angle

### Subscripts

$c$  = Cold wall  
 $f$  = Fluid  
 $h$  = Hot wall  
 $s$  = Solid insulation

0961-5539/96

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Received May 1995

Revised July 1995

## INTRODUCTION

Air spaces are often filled with either porous or non-porous thermal insulating materials to suppress convection, thereby reducing heat losses. A common example is the insulation of air spaces in the walls of buildings. The present numerical study examines the effect of partially filling an air space with a non-porous insulation. Emphasis is placed on determining the conditions that minimize the overall heat transfer rate. From an engineering perspective, it may be possible to optimize thermal insulation usage from such data.

Several previous numerical studies have considered enclosures partly filled with a porous thermal insulating material. A partly filled vertical enclosure with no impermeable barrier between the fluid and porous insulation has been studied by Oosthuizen and Paul<sup>1</sup>, Song and Viskanta<sup>2</sup>, Arquis *et al.*<sup>3</sup> and Beckermann *et al.*<sup>4</sup>. In addition, Tong and Subramanian<sup>5</sup> have presented data for a partially filled vertical enclosure with an impermeable barrier between the fluid and porous medium. The effect of angle of inclination on the heat transfer in an enclosure partly filled with porous medium (with a barrier) was studied numerically by Oosthuizen and Paul<sup>6</sup>. In these numerical studies, the porous (fibrous or particulate) insulation has been modelled as a saturated porous medium.

Several of the above studies have shown that under certain conditions it is possible to obtain almost the same overall heat transfer rate with a partly filled enclosure as for a completely filled enclosure. In fact, Tong and Subramanian<sup>5</sup> predicted that for some conditions, the minimum overall heat transfer rate can occur when a vertical enclosure is only about 60 per cent filled with porous insulation. This numerical prediction was later verified experimentally for an enclosure partly filled with a porous medium saturated with water<sup>7</sup>. However, similar experiments performed with porous insulations in air-filled enclosures have not shown this behaviour. Vafai and Belwafa<sup>8</sup> found experimentally that the heat transfer rate in a vertical enclosure increased as soon as an air gap was introduced, even for insulation-to-air conductivity ratios as high as 3.3. Experimental results obtained for a horizontal enclosure heated from below<sup>9</sup> also showed that for air, a completely filled enclosure has the minimum heat transfer rate.

In contrast to the above referenced studies, the current work focuses on an enclosure partially filled with a non-porous insulating material. The model geometry and co-ordinate system of the problem is shown in *Figure 1*. A rectangular enclosure of height  $H$  and width  $W$  has one wall heated to temperature  $T_h$  and the other vertical wall maintained at temperature  $T_c$ . The enclosure is inclined at an angle  $\Phi$  with respect to the horizontal and is partly filled with a finitely conducting non-porous solid of thickness  $S$ , adjacent to the heated surface. In this study, the upper and lower surfaces are assumed to be either adiabatic or perfectly conducting. Although neither ideal boundary condition can be obtained in practice, the free convection heat transfer rate in an enclosure with finitely conducting end walls (i.e. in a real enclosure) has been shown to be between these two limiting cases (Oosthuizen and Paul<sup>10</sup>).

The main purpose of the current study is to investigate the convective heat transfer in an enclosure partially filled with solid insulation. Emphasis is placed on determining the conditions that minimize the overall heat transfer rate. Of course, the results of the present study will be the same as for an enclosure partly filled with a porous medium for a Darcy number approaching zero.

Similar conjugate problems in vertical enclosures have been studied by Kaminski and Prakash<sup>11</sup> and Lauriat<sup>12</sup>. However, in these studies the thickness of the fluid layer (and hence the aspect ratio of the fluid-filled cavity) was held constant and only the solid wall thickness was varied. For this reason the results from these studies are not directly applicable to the current "optimal insulation" problem.

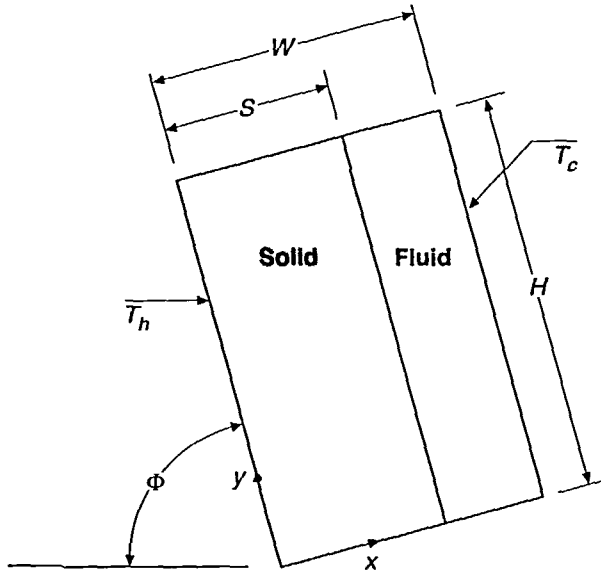


Figure 1 The model geometry

### GOVERNING EQUATIONS AND SOLUTION PROCEDURE

The flow is assumed to be steady, laminar, incompressible and two-dimensional. The thermophysical properties of the fluid and solid are assumed to be constant, except for fluid density which is treated by means of the Boussinesq approximation. Radiation heat exchange between the surfaces in the enclosure has been neglected in comparison with convection, and the fluid is assumed to be radiatively non-participating. With these assumptions, the governing equations for the fluid are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + g\beta(T - T_c) \cos \Phi \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta(T - T_c) \sin \Phi \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_f}{\rho C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

The steady two-dimensional conduction in the solid wall is governed by Laplace's equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (5)$$

In the fluid layer, the solution has been obtained in terms of stream function ( $\psi$ ) and vorticity ( $\omega$ ), defined as:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (6)$$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \quad (7)$$

Also, the following dimensionless variables have been defined:

$$X = \frac{x}{W}, \quad Y = \frac{y}{W}, \quad \theta = \frac{(T - T_c)}{(T_h - T_c)} \quad (8)$$

$$\Psi = \frac{\psi Pr}{\nu}, \quad \Omega = \frac{\omega W^2 Pr}{\nu}. \quad (9)$$

In terms of these variables, the dimensionless governing equations for the fluid layer become:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\Omega \quad (10)$$

$$\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} = \frac{1}{Pr} \left( \frac{\partial \Psi}{\partial Y} \frac{\partial \Omega}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \Omega}{\partial Y} \right) + Ra \left( \frac{\partial \theta}{\partial Y} \cos \Phi - \frac{\partial \theta}{\partial X} \sin \Phi \right) \quad (11)$$

$$\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = \frac{\partial \Psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \theta}{\partial Y} \quad (12)$$

where  $Pr$  is the fluid Prandtl number and the Rayleigh number is:

$$Ra = \frac{g\beta(T_h - T_c)W^3}{\nu\alpha}. \quad (13)$$

In the solid wall, the dimensionless conduction equation is:

$$\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = 0 \quad (14)$$

The temperature boundary conditions on the outside of the enclosure are:

$$\text{For } X=0, 0 \leq Y \leq A \quad \theta = 1 \quad (15)$$

$$\text{For } X=1, 0 \leq Y \leq A \quad \theta = 0 \quad (16)$$

$$\text{For } Y=0, A \leq X \leq 1 \quad \frac{\partial \theta}{\partial Y} = 0, \text{ or } \theta = 1 - X \quad (17)$$

where  $A = H/W$  is the enclosure aspect ratio. Continuity conditions for temperature and heat flux are applied at the solid-fluid interface:

$$\text{For } X = S/W, 0 \leq Y \leq A \quad \theta_{\text{fluid}} = \theta_{\text{solid}}, \quad \left( \frac{\partial \theta}{\partial X} \right)_{\text{fluid}} = k_r \left( \frac{\partial \theta}{\partial X} \right)_{\text{solid}} \quad (18)$$

where  $k_r = k_s/k_f$  is the solid-to-fluid conductivity ratio. No slip and impermeability conditions are applied at all the bounding surfaces of the fluid layer:

$$\text{For } X = S/W, 1 \quad 0 \leq Y \leq A \quad \Psi = 0 \quad \frac{\partial \Psi}{\partial X} = 0 \quad (19)$$

$$\text{For } Y = 0, A \quad S/W \leq X \leq 1, \quad \Psi = 0 \quad \frac{\partial \Psi}{\partial Y} = 0. \quad (20)$$

The dimensionless governing equations (10)-(14) have been solved subject to boundary conditions (15)-(20) using a Galerkin finite element method. Triangular elements with linear interpolation functions were used. The discretized equations were solved iteratively using successive over-relaxation.

The solution directly gives the average heat transfer rate on the hot and cold walls. The hot and cold wall average Nusselt numbers are defined as:

$$Nu_h = \frac{Q_h}{k_f(T_h - T_c)A}, \quad Nu_c = \frac{Q_c}{k_f(T_h - T_c)A} \quad (21)$$

where  $Q_h$  is the total heat transfer rate from the hot wall and  $Q_c$  is the total heat transfer rate to the cold wall. With these definitions, the Nusselt numbers can be interpreted as the ratio of the total heat transfer rate ( $Q_h$  or  $Q_c$ ) to the heat transfer rate by pure conduction when the entire enclosure is filled with fluid alone. Note that in general, for perfectly conducting end wall boundary conditions, the hot and cold wall average Nusselt numbers will be different (except for  $S/W = 0, 1$ ), because of heat losses or gains through the end walls.

Most of the calculations were done for an aspect ratio of  $A = 5$  with a  $(13 + 27) \times 105$  non-uniform mesh (with 4,200 nodes, 7,904 elements). The notation  $(13 + 27) \times 105$  indicates that in the  $X$ -direction there were 13 nodes in the solid and 27 nodes in the fluid and in the  $Y$ -direction there were 105 nodes. Grid testing was performed for a range of parameters and the average Nusselt number data were found to be grid independent to better than 1 per cent; some sample results are shown in Table 1. Also, comparisons have been made with a closely related numerical solution of conjugate-free convection in an enclosure by Kaminski and Prakash<sup>11</sup>. In Table 2 the average Nusselt number data of Kaminski and Prakash are shown in parentheses following the values predicted by the present solution. The results are in close agreement.

Table 1 Effect of mesh size on average Nusselt number for  $Ra = 10^5$ ,  $A = 5$ ,  $k_r = 1.5$ ,  $Pr = 0.7$ , adiabatic end walls

$S/W$	Mesh size	$Nu_h$
0.4	$(13 + 27) \times 105$	1.774
0.4	$(19 + 39) \times 141$	1.772
0.6	$(13 + 27) \times 105$	1.377
0.6	$(19 + 39) \times 141$	1.377
0.8	$(13 + 27) \times 105$	1.364
0.8	$(19 + 39) \times 141$	1.364

Table 2 Average Nusselt number data from the current solution and the solution of Kaminski and Prakash<sup>11</sup> (in parentheses) for  $Gr = 10^5$  based on  $(W-S)$ ,  $Pr = 0.7$ ,  $(W-S)/H = 1.0$ ,  $\Phi = 90^\circ$ , adiabatic end walls

Conductance ratio				
$k_r (W-S)/S$	$S/(W-S) = 0.2$		$S/(W-S) = 0.4$	
5	2.08	(2.08)	2.08	(2.08)
25	3.44	(3.42)	3.43	(3.41)
50	3.72	(3.72)	3.75	(3.71)

### THERMAL RESISTANCE MODEL

A one-dimensional thermal resistance model can also be used to estimate the overall heat transfer rate. With this approach the average Nusselt number can be easily expressed as:

$$Nu_h = Nu_c = \left( \frac{S/W}{k_r} + \frac{1-S/W}{Nu_{cor}} \right)^{-1} \quad (22)$$

where  $Nu_{cor} = h(W-S)/k_f$ .  $Nu_{cor}$  is the average Nusselt number obtained from an empirical correlation for free convection in a fluid-filled enclosure with wall spacing  $(W-S)$ . In this method the solid-fluid interface is assumed to be at a uniform dimensionless temperature  $\theta_i$ , which can be expressed as:

$$\theta_i = \left[ 1 + Nu_{cor} \left( \frac{S/W}{k_r(1-S/W)} \right) \right]^{-1} \quad (23)$$

There are many empirical correlations in the literature for the average Nusselt number in a vertical enclosure ( $Nu_{cor}$ ). However, most of these correlations cannot be applied to the wide range of fluid enclosure aspect ratios encountered in the current problem. For this reason, only the following correlation, recommended by ElSherbiny *et al.*<sup>13</sup>, has been used to evaluate the Nusselt number in the fluid gap:

$$\begin{aligned}
 Nu_1 &= 0.0605 Ra_{cor}^{\frac{1}{3}} \\
 Nu_2 &= \left[ 1 + \left( \frac{0.104 Ra_{cor}^{0.293}}{1 + (6310/Ra_{cor})^{1.36}} \right)^3 \right]^{\frac{1}{3}} \\
 Nu_3 &= 0.242 \left( \frac{Ra_{cor}}{A_{cor}} \right)^{0.272} \\
 Nu_{cor} &= \text{Max}(Nu_1, Nu_2, Nu_3).
 \end{aligned} \quad (24)$$

As indicated by the last equation,  $Nu_{cor}$  is the maximum value of  $Nu_1$ ,  $Nu_2$  and  $Nu_3$ . This correlation fits an extensive set of experimental data over a wide range of aspect ratios ( $5 \leq A_{cor} \leq 110$ ) and Rayleigh number ( $10 \leq Ra_{cor} \leq 2 \times 10^7$ ) with a maximum difference of 9 per cent. In

equation (24), the correlation Rayleigh number ( $Ra_{cor}$ ) is related to the current Rayleigh number definition ( $Ra$ ) by:

$$Ra_{cor} = Ra(1 - S/W)^3 \theta_t . \tag{25}$$

Also, the correlation aspect ratio ( $A_{cor}$ ) in equation (24) is:

$$A_{cor} = A/(1 - S/W) . \tag{26}$$

Equations (23) and (24) must be solved iteratively to obtain  $Nu_{cor}$  and  $\theta_t$ . Once  $Nu_{cor}$  has been obtained, the overall Nusselt number can be calculated from equation (22).

### RESULTS

The solution to the conjugate problem is governed by six dimensionless parameters:

- (1) the Rayleigh number  $Ra$ ;
- (2) the fluid Prandtl number  $Pr$ ;
- (3) the enclosure aspect ratio  $A$ ;
- (4) the enclosure inclination angle  $\Phi$ ;
- (5) the dimensionless insulation thickness  $S/W$ ; and
- (6) the solid-to-fluid conductivity ratio  $k_r$ .

The range of parameters considered in this study is  $0 \leq Ra \leq 10^6$ ,  $2 \leq A \leq 10$ ,  $30^\circ \leq \Phi \leq 180^\circ$ ,  $0 \leq S/W \leq 1$  and  $1 \leq k_r \leq 2$ . Solutions have been obtained for only  $Pr = 0.7$  because the most likely applications are for air. Note that for applications involving air,  $k_r = 1$  represents a lower limit for practical building insulation materials.

Figure 2 shows the effect of solid insulation thickness on the heat transfer rate in a vertical enclosure with adiabatic end walls. The average Nusselt number variation predicted by the present two-dimensional numerical solution and the one-dimensional thermal resistance model are shown for  $k_r = 1, 1.5$  and  $2$ . It can be seen that a simple thermal resistance model yields average Nusselt numbers about 5-8 per cent lower than the two-dimensional numerical solution. This level of agreement is within the uncertainty of the correlation (Equation (24)) used to calculate  $Nu_{cor}$ .

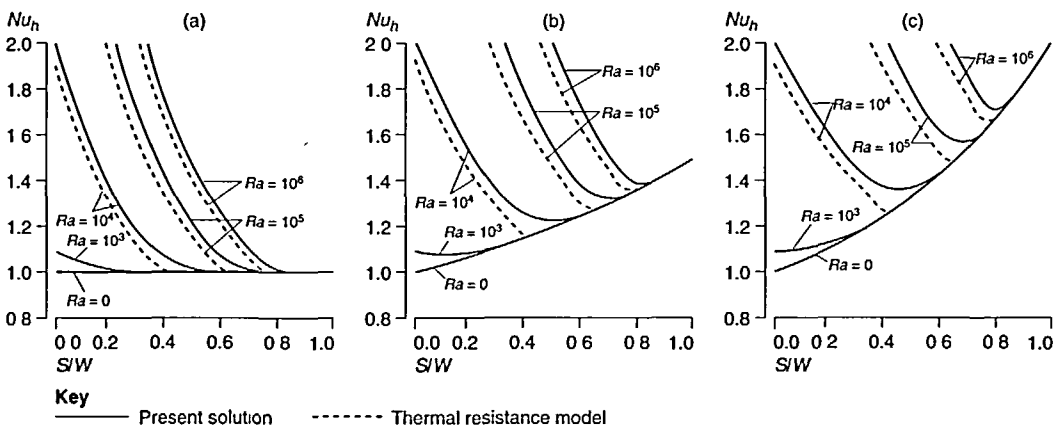
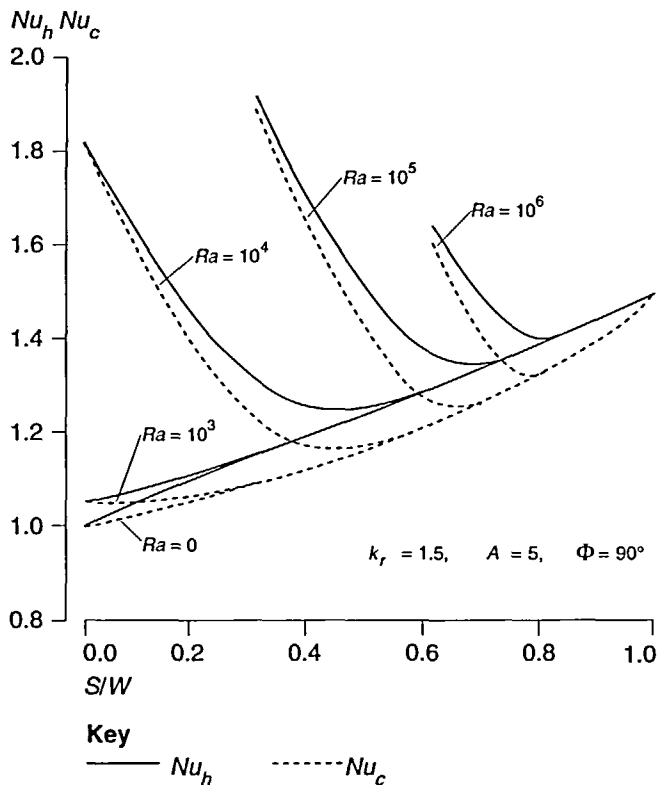


Figure 2 The effect of solid insulation thickness ( $S/W$ ) on the average Nusselt number for a vertical enclosure ( $A = 5$ ) with adiabatic side walls (a)  $k_r = 1$ , (b)  $k_r = 1.5$ , (c)  $k_r = 2$

Referring to *Figure 2*, in each case the maximum Nusselt number corresponds to a fluid-filled enclosure ( $S/W = 0$ ). As the insulation thickness is increased, the Nusselt number initially decreases because of weakening convection in the air layer. The convection weakens because of the narrowing air gap and the decreasing temperature difference across the gap. Eventually, as more insulation is added, the air layer becomes almost stagnant and the heat transfer occurs almost by pure conduction. Hence, for  $k_r = 1$  (*Figure 2(a)*) the Nusselt number reaches the pure conduction limit ( $Nu_h = 1$ ) and remains constant. However, for conductivity ratios greater than one (*Figures 2(b) and (c)*), a minimum average Nusselt number occurs when the enclosure is only partly filled. The minimum heat transfer rate occurs when the convective motion in the air layer is reduced to the extent that its effective thermal conductivity is equal to the conductivity of the solid insulation. For  $S/W$  greater than this "optimum" thickness the Nusselt number increases because the air layer has a lower effective conductivity than the solid insulation. Comparing *Figures 2(a)*, *(b)*, and *(c)* it can be seen that for a fixed Rayleigh number, the optimum insulation thickness decreases as the conductivity ratio increases. Also, it is evident that the thermal resistance method can be used to estimate the optimum insulation thickness. This one-dimensional method gives optimum thicknesses that are only slightly lower than those obtained from the current two-dimensional conjugate solution.

It can also be seen in *Figure 2* that the optimum thickness is strongly depended on Rayleigh number. As the Rayleigh number increases, the optimum insulation thickness (for a fixed

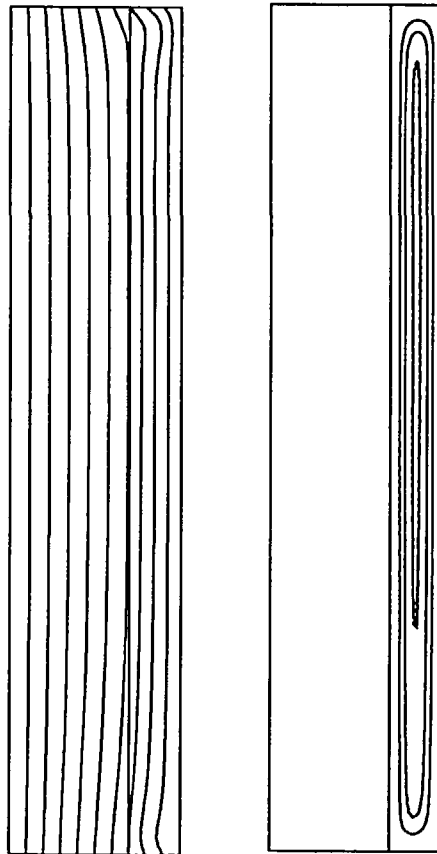


*Figure 3* The effect of perfectly conducting (PC) side walls on the average Nusselt numbers in a partially filled vertical enclosure ( $k_r = 1.5$ ,  $A = 5$ )



conductivity ratio) also increases. This may explain, in part, the experimental findings of Vafai and Belwafa<sup>8</sup>. They found that for a Rayleigh number range of  $7.5 \times 10^5 \leq Ra \leq 1.8 \times 10^6$ , the heat transfer rate in a vertical enclosure increased as soon as an air gap was introduced. It can be seen from *Figure 2* that in this  $Ra$  range, the minimum Nusselt number for a partly filled enclosure is only slightly lower than for a completely filled enclosure and may be difficult to detect in a heat balance experiment. It should also be noted that Vafai and Belwafa<sup>8</sup> included radiation heat transfer in their data. Radiation will tend to offset the reduction in the conduction heat transfer achieved by partially filling the enclosure. However, the current results represent the behaviour that could be approached with the used low emissivity cladding on the insulation

*Figure 3* shows the effect of perfectly conducting (PC) end walls on the heat transfer rate in a partially filled vertical enclosure ( $k_r = 1.5$ ). Note that the hot wall average Nusselt number is higher than the cold wall average Nusselt number because of heat transfer to the end walls. *Figure 4* shows the isotherms for an insulation thickness ( $S/W = 0.7$ ) that is close to the optimum for  $Ra = 10^5$ ,  $\Phi = 90^\circ$ ,  $A = 5$  and  $k_r = 1.5$ . It can be seen from the temperature contours that the difference in  $Nu_h$  and  $Nu_c$  can be attributed primarily to heat transfer from the solid and fluid to the upper end wall.



*Figure 4* Temperature and streamline contours for a vertical enclosure partially filled with a solid insulation and PC side walls,  $S/W = 0.7$ ,  $Ra = 10^5$ , and  $k_r = 1.5$ ,  $\Psi_{\max} = -2.7$  ( $\Delta T = 0.1$ ,  $\Delta \Psi = 0.8$ )

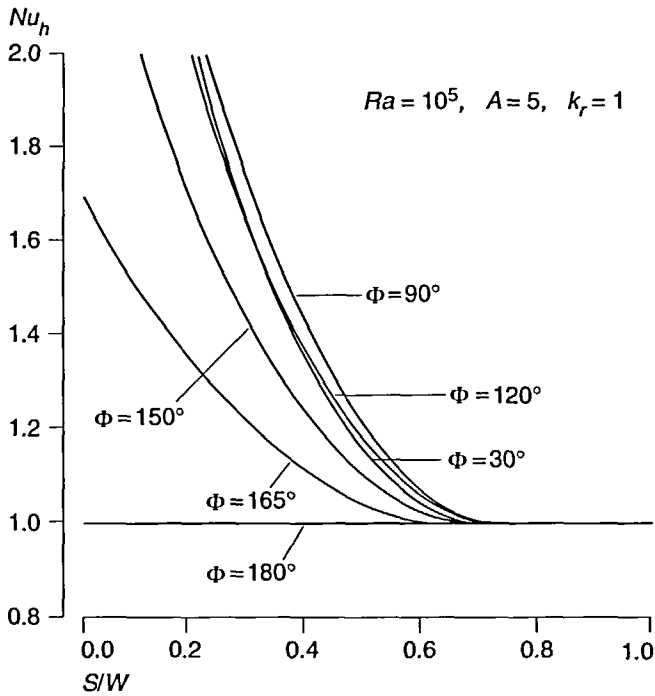


Figure 5 The effect of the enclosure inclination angle ( $\Phi$ ) on the average Nusselt number in a partially filled enclosure with adiabatic side walls ( $A = 5$ ,  $Ra = 10^5$ , and  $k_r = 1$ )

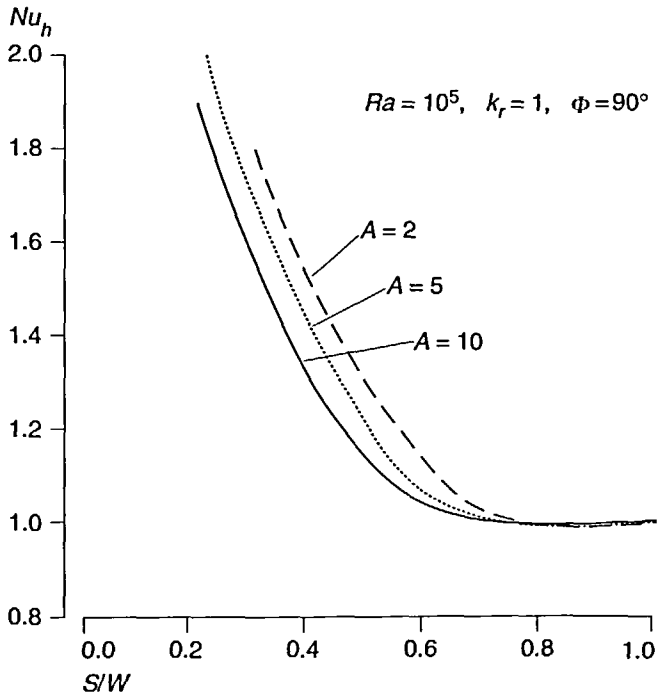


Figure 6 The effect of the enclosure aspect ratio ( $A$ ) on the average Nusselt number in a partially filled vertical enclosure with adiabatic side walls,  $Ra = 10^5$ , and  $k_r = 1$

In *Figure 3* it is shown that the minima in the  $Nu_h$  and  $Nu_c$  curves occur at approximately the same insulation thickness. However,  $Nu_c$  is substantially lower than  $Nu_h$  at the optimum thickness. Also, comparison of *Figure 3* and *Figure 2(b)* shows that the value of  $Nu_h$  at the optimum thickness is higher than for the adiabatic end wall case. As might be expected, results for lower aspect ratio enclosures with PC end walls (not shown) displayed stronger wall conduction effects. For  $A = 2$  and  $k_r = 1.5$  there was almost no reduction in  $Nu_h$  for a partly filled vertical enclosure compared with the completely filled enclosure at  $Ra = 10^5$ .

Again, the above results may partially explain the experimental data of Vafai and Belwafa<sup>8</sup> for an insulated wall space. Their experimental enclosure had plywood end walls and the heat transfer rate from the hot wall ( $Nu_h$ ) was determined from a heat balance. The current results show that in a practical wall configuration the reduction in the heat transfer rate achieved by partially filling the enclosure will be offset to some extent by additional end wall conduction. Of course, wall conduction effects in a practical enclosure may be substantially smaller than for PC end walls.

The effect of the enclosure inclination angle ( $\Phi$ ) on the average Nusselt number in a partially filled vertical enclosure ( $k_r = 1$ ) with adiabatic end walls is shown in *Figure 5*. It can be seen that the minimum insulation thickness required to produce essentially pure conduction heat transfer decreases only slightly for moderate inclination angles. Hence, the optimum insulation thickness for conductivity ratios ( $k_r$ ) greater than one can be expected to be insensitive to moderate enclosure inclination.

*Figure 6* shows the effect of the enclosure aspect ratio ( $A = H/W$ ) on the average Nusselt number in a partially filled vertical enclosure ( $k_r = 1$ ) with adiabatic end walls. It can be seen that the minimum insulation thickness required to produce essentially pure conduction heat transfer decreases only slightly as the aspect ratio increases.

## SUMMARY

Laminar, two-dimensional, conjugate natural convection in an enclosure partially filled with a finitely conducting solid has been solved numerically. The emphasis of the study was to examine the conditions that produce the minimum overall heat transfer rate.

The results indicate that, for solid-to-fluid conductivity ratios greater than one, the minimum overall heat transfer rate occurs for a partially filled enclosure. Also, it has been shown that a one-dimensional thermal resistance model can be used to estimate this optimum thermal insulation thickness. However, in practical configurations, the reduction in the heat transfer rate achieved by partially filling the enclosure will be offset to some extent by additional end wall conduction (and radiation).

## ACKNOWLEDGEMENTS

This work was supported by the Natural Sciences and Engineering Research Council of Canada.

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